

Work & Energy

October 28, 2024

Introduction

- A system is any group of objects.
- A system has a total energy E , the sum of the different forms of energy (kinetic, gravitational, electrical, thermal, chemical, ...)
- Conservation of Energy: Energy is never created or destroyed, only transferred between systems, or between different forms. (ex. friction converts kinetic energy to heat)
- Energy is a scalar quantity, measured in Joules (J)

Work

- Work is the transfer of mechanical energy.
- If a constant force F moves an object through a displacement d in the direction of the force, the work W is defined as

$$W = Fd$$

- If the force is not parallel to the displacement, take the parallel component:

$$W = \mathbf{F} \cdot \mathbf{d} = Fd \cos \theta$$

- Note that this is signed, if the force is in the opposite direction of motion, the work is negative
- Unit of work is Joules, same as energy
- If the force is variable, we simply integrate small displacements with $dW = \mathbf{F} \cdot d\mathbf{x}$, which gives

$$W = \int \mathbf{F} \cdot d\mathbf{x}.$$

- By Newton's third law, the work done *on* an object is negative the work done *by* the object.

Practice Problems

1. A block is pulled by a string with tension 10 N at an angle of 60° with the horizontal, across a distance of 10 m, what is the work done on the block.
2. An object of mass 2.0 kg moves in uniform circular motion with radius 0.75 m on a horizontal frictionless table. If the centripetal force is 10.0 N, what is the work done by this force when the object moves through one complete revolution?
3. A uniform block of mass 10 kg is released at rest from the top of an incline with length 10 m and inclination 30° , and slides to the bottom. The coefficients of static and kinetic friction are $\mu_s = \mu_k = 0.1$. How much energy is dissipated due to friction?
 - (A) 0 J
 - (B) 22 J
 - (C) 43 J
 - (D) 87 J
 - (E) 164 J

Power

- Power is the rate at which work is done
- It is defined as

$$P = \frac{dE}{dt} = \frac{Fdx}{dt} = Fv$$

- Unit of power is Joules/seconds, or Watts

Kinetic Energy

- Translation kinetic energy: energy of motion

$$KE = \frac{1}{2}mv^2$$

- Work-Kinetic Energy Theorem: The work done by the net force on a particle equals its change in kinetic energy

$$W_{net} = \Delta KE$$

- Let a force F accelerate a mass m for a distance d . We have $W = Fd = mad$ By kinematics $v^2 = 2ad$, so $\Delta KE = W = \frac{1}{2}mv^2$

Practice Problems

1. A 0.20-kg puck slides across a frictionless floor with a speed of 10m/s. The puck strikes a soft wall and stops. What is the net work done on the puck?
2. A 0.20-kg puck slides across a frictionless floor with a speed of 10m/s. The puck strikes a soft wall and bounces off with a speed of 10m/2 in the opposite direction. What is the net work done on the puck?

Conservative and Nonconservative forces

- The work done by a conservative force does not depend on the path taken, only on the initial and final positions. (ex. gravity)
- The work done by a nonconservative force depends on the path. Typical nonconservative forces are external forces, or friction.
- Conservative forces conserve mechanical energy.

Potential Energy

- We can define a potential energy for any *conservative* force.
- Potential Energy is energy stored in a system. In mechanics, we will focus mainly on gravitational potential energy and elastic potential energy (springs).
- When the gravity force lifts up an object a distance h , it does work $W = Fd = mgh$. Thus, it stores gravitational potential energy

$$U_g = mgh$$

- For potential energy, we can take the height from any reference point; we only care about changes in the potential energy

$$\Delta U_g = mg\Delta h$$

Conservation of Energy

Usually in mechanics, we only consider kinetic and potential energies. If a system is isolated, there is no external force, so $\Delta E = W_{nc} = 0$. Thus, the energy is conserved and constant. We can write this as

$$E_1 = E_2$$

$$K_1 + U_1 = K_2 + U_2$$

If the potential energy is purely gravitational, this is just

$$\frac{1}{2}m_1v_1^2 + m_1gh_1 = \frac{1}{2}m_2v_2^2 + m_2gh_2$$

If the system is not isolated, then there is an external force, and the work done by the external is the change in the total energy.

Practice Problems

1. A ball of mass m is dropped from rest from a height h . Calculate the velocity right before it reaches the ground, using conservation of energy. Compare your answer to kinematics.

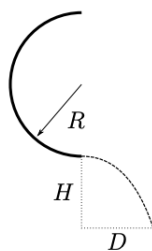
2.

Three stones A, B, and C are thrown with the same initial speed at the same instant from the roof of a building with angles of 60° , 30° , and 0° above the horizontal, respectively. Which statement below is true? (Neglect air resistance.)

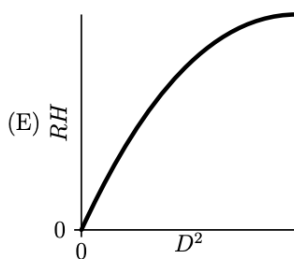
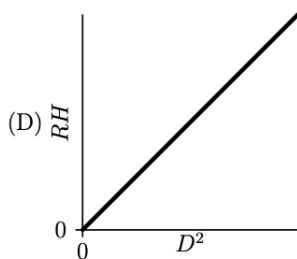
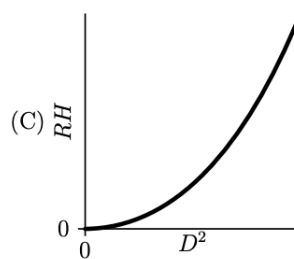
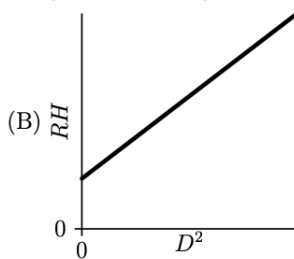
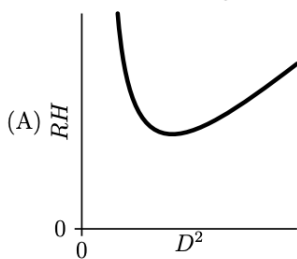
- The stones strike the ground at the same time with equal speeds $v_A = v_B = v_C$.
- The stones strike the ground at the same time with speeds $v_A > v_B > v_C$.
- The stones strike the ground at equal speeds with times in the air $t_B > t_A > t_C$.
- The stones strike the ground at equal speeds with times in the air $t_A > t_B > t_C$.
- The stones strike the ground at different times with different speeds.

3.

A semicircular wire of radius R is oriented vertically. A small bead is released from rest at the top of the wire, it slides without friction under the influence of gravity to the bottom, where it then leaves the wire horizontally and falls a distance H to the ground. The bead lands a horizontal distance D away from where it was launched.



Which of the following is a correct graph of RH against D^2 ?



4.

A packing crate with mass $m = 115 \text{ kg}$ is slid up a 5.00 m long ramp which makes an angle of 20.0° with respect to the horizontal by an applied force of $F = 1.00 \times 10^3 \text{ N}$ directed parallel to the ramp's incline. A frictional force of magnitude $f = 4.00 \times 10^2 \text{ N}$ resists the motion. If the crate starts from rest, what is its speed at the top of the ramp?

- (A) 4.24 m/s
- (B) 5.11 m/s
- (C) 7.22 m/s
- (D) 8.26 m/s
- (E) 9.33 m/s

Work-Energy relationships

By the Work-Kinetic Energy Theorem

$$W_{net} = \Delta K$$

Also, the work done by any nonconservative or external forces equals the change in the total energy (mechanical) of a system:

$$W_{nc} = \Delta E$$

Since the total energy is the sum of the kinetic and potential energies $E = K + U$, we get that the work done by conservative forces is negative the change in potential energy

$$W_c = -\Delta U$$

(i.e. conservative forces convert from potential energy to energy)

1. A ball of mass 10 kg is dropped at a height of 4 m from one side of a U-shaped half pipe. If friction does 35 J of work on the ball What is the maximum height it can reach when it slides up the other side?

2.

A block of mass m , starting from rest, is pulled up a frictionless inclined plane that makes an angle θ with the horizontal by a string parallel to the plane. The tension in the string T is not constant but depends on the distance travelled. After traveling a distance L , the speed of the block is v . The work done by the tension is

- a. $mgL \sin \theta$ b. $mgL \cos \theta + \frac{1}{2} m v^2$ c. $mgL \sin \theta + \frac{1}{2} m v^2$ d. $\frac{1}{2} m v^2$

e. undetermined without knowing the dependence of T on the distance.

3.

18. A block of mass $m = 3.0 \text{ kg}$ slides down one ramp, and then up a second ramp. The coefficient of kinetic friction between the block and each ramp is $\mu_k = 0.40$. The block begins at a height $h_1 = 1.0 \text{ m}$ above the horizontal. Both ramps are at a 30° incline above the horizontal. To what height above the horizontal does the block rise on the second ramp?

- (A) 0.18 m
- (B) 0.52 m
- (C) 0.59 m
- (D) 0.69 m
- (E) 0.71 m

4. (harder) A block is given an initial velocity 30 m/s up a ramp of angle 45° . The coefficient of friction is 0.5. What is the maximum height the block can reach?

Energy diagrams

Recall for variable forces, the work is given by

$$W = \int \mathbf{F} \cdot d\mathbf{x}.$$

Thus, if given a function of force with respect to position, the work done is simply the area under the curve.

Also, recall that

$$W_c = -\Delta U$$

If we consider a small displacement, this is just $Fdx = -dU$ or

$$F = -\frac{dU}{dx}.$$

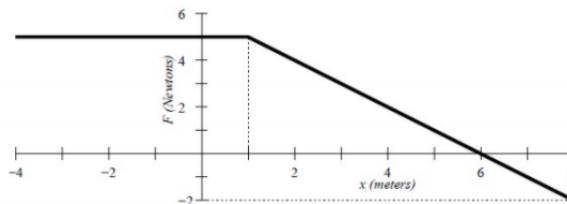
If given a function of potential energy with respect to position, the force (by conservative forces) is simply the slope of the curve. This means the force is in the direction of lower energy. As a result, a particle will in a stable equilibrium (no net force) when the potential energy is at a minimum (i.e. a system always wants to minimize its potential energy)

- Consider a ball lying on the graph, since $U = mgh \propto h$. The ball will fall towards lower altitudes.

Practice Problems

1.

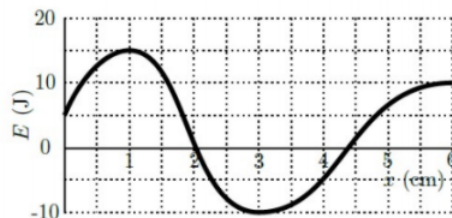
Shown below is a graph of the x component of force versus position for a 4.0 kg cart constrained to move in one dimension on the x axis. At $x = 0$ the cart has a velocity of -3.0 m/s (in the negative direction). Which of the following is closest to the maximum speed of the cart?



- (A) 1.6 m/s
- (B) 2.5 m/s
- (C) 3.0 m/s
- (D) 4.0 m/s
- (E) 4.2 m/s

2.

Shown below is a graph of potential energy as a function of position for a 0.50 kg object.

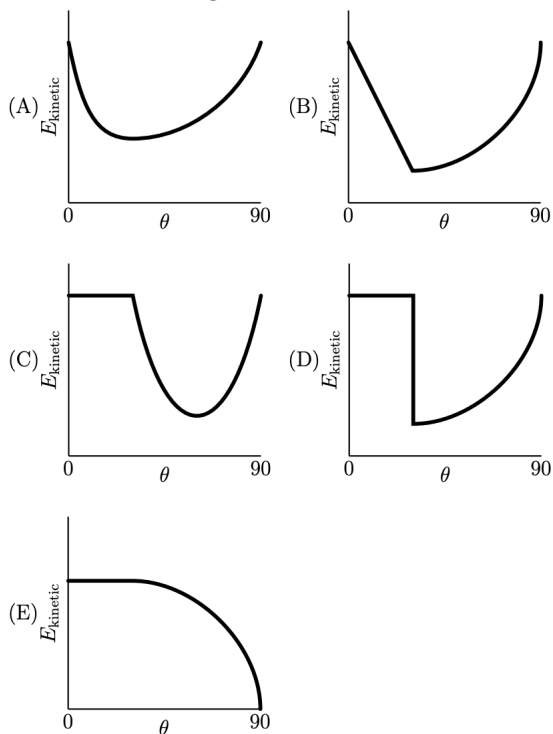


Which of the following statements is NOT true in the range $0 \text{ cm} < x < 6 \text{ cm}$?

- (A) The object could be at equilibrium at either $x = 1 \text{ cm}$ or $x = 3 \text{ cm}$.
- (B) The minimum possible total energy for this object in the range is -10 J .
- (C) The magnitude of the force on the object at 4 cm is approximately 1000 N .
- (D) If the total energy of the particle is 0 J then the object will have a maximum kinetic energy of 10 J .
- (E) The magnitude of the acceleration of the object at $x = 2 \text{ cm}$ is approximately 4 cm/s^2 .

3.

A solid ball is released from rest down inclines of various inclination angles θ but through a fixed vertical height h . The coefficient of static and kinetic friction are both equal to μ . Which of the following graphs best represents the total kinetic energy of the ball at the bottom of the incline as a function of the angle of the incline?



Extended Problems

Question 1

The chemical potential energy stored in a battery is converted into kinetic energy in a toy car that increases its speed first from 0 mph to 2 mph and then from 2 mph up to 4 mph. Ignore the energy transferred to thermal energy due to friction and air resistance. Compared to the energy required to go from 0 to 2 mph, the energy required to go from 2 to 4 mph is

- (a) half the amount.
- (b) the same amount.
- (c) twice the amount.
- (d) three times the amount.
- (e) four times the amount.

Question 2

Batman, who has a mass of $M = 100$ kg, climbs to the roof of a 30 m building and then lowers one end of a massless rope to his sidekick Robin. Batman then pulls Robin, who has a mass of $m = 75$ kg, up the roof of the building. Approximately how much total work has Batman done after Robin is on the roof?

- (A) 60 J
- (B) 7×10^3 J
- (C) 5×10^4 J
- (D) 600 J
- (E) 3×10^4 J

Question 3

A point-like mass moves horizontally between two walls on a frictionless surface with initial kinetic energy E . With every collision with the walls, the mass loses $\frac{1}{2}$ its kinetic energy to thermal energy. How many collisions with the walls are necessary before the speed of the mass is reduced by a factor of 8?

- (a) 3
- (b) 4
- (c) 6
- (d) 8
- (e) 16

Question 4

A block with mass m starts from rest and slides down a plane inclined at an angle θ . The coefficient of kinetic friction is μ . Which expression correctly yields the block's speed v after it has traveled a distance d down along the plane, assuming that it does indeed start sliding down? (d is a distance here, so it is a positive quantity.)

(a) $mgd \sin \theta + \mu mgd \cos \theta = \frac{mv^2}{2}$

(b) $mgd \sin \theta - \mu mgd \cos \theta = \frac{mv^2}{2}$

(c) $-mgd \sin \theta + \mu mgd \cos \theta = \frac{mv^2}{2}$

(d) $mgd \cos \theta + \mu mgd \sin \theta = \frac{mv^2}{2}$

(e) $mgd \cos \theta - \mu mgd \sin \theta = \frac{mv^2}{2}$

Question 5

A car has an engine which delivers a constant power. It accelerates from rest at time $t = 0$, and at $t = t_0$ its acceleration is a_0 . What is its acceleration at $t = 2t_0$? Ignore energy loss due to friction.

(a) $\frac{1}{2}a_0$

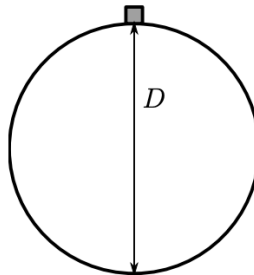
(b) $\frac{1}{\sqrt{2}}a_0$

(c) a_0

(d) $\sqrt{2}a_0$

(e) $2a_0$

A small bead is placed on the top of a frictionless glass sphere of diameter D as shown. The bead is given a slight push and starts sliding down along the sphere. Find the speed v of the bead at the point at which the bead leaves the sphere.



- (A) $v = \sqrt{gD}$
- (B) $v = \sqrt{4gD/5}$
- (C) $v = \sqrt{2gD/3}$
- (D) $v = \sqrt{gD/2}$
- (E) $v = \sqrt{gD/3}$

A small ball of mass $3m$ is at rest on the ground. A second small ball of mass m is positioned above the ground by a vertical massless rod of length L that is also attached to the ball on the ground. The original orientation of the rod is directly vertical, and the top ball is given a small horizontal nudge. There is no friction; assume that everything happens in a single plane.

Determine the speed v of the second (originally top) ball just before it hits the ground.

- (A) $v = \sqrt{2gL}$
- (B) $v = \sqrt{gL}$
- (C) $v = \sqrt{2gL/3}$
- (D) $v = \sqrt{3gL/2}$
- (E) $v = \sqrt{gL/4}$