

Kinematics

September 8, 2025

Kinematics

I. General Definitions:

Average quantities:

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

Instantaneous quantities:

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

II. Five Kinematics Equations: To use, acceleration must be constant: $a = a_{avg} = \frac{\Delta v}{\Delta t}$

1. $v_1 = v_0 + at$

2. $\Delta x = \frac{1}{2}(v_0 + v)t$

3. $v_1^2 = v_0^2 + 2a\Delta x$

4. $\Delta x = v_0t + \frac{1}{2}at^2$

5. $\Delta x = v_ft - \frac{1}{2}at^2$

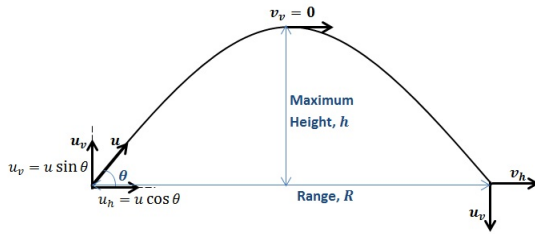
Practice Problems

1. The position of a particle is given by $x(t) = 3.0t + 0.5t^3m$. (a) Find the instantaneous velocity at $t = 2.0s$. (b) Calculate the average velocity between $t = 1.0 s$ and $t = 3.0 s$.

2. Beaker rolls a bowling ball up a slope at $2.0 m/s$. It travels $2.7 m$, comes to a stop and rolls back down. What is the bowling ball's entire time of travel?

III. Projectile Motion:

The simplest case of projectile motion: $t_{up} = t_{down}$, $v_{up} = v_{down}$



Consider an object projected with an initial velocity v_0 at an angle θ with the horizontal surface. The components of initial velocity, in the x and y directions respectively, are:

$$v_{0x} = v_0 \cos \theta$$

$$v_{0y} = v_0 \sin \theta$$

In the absence of air resistance, the projectile experiences constant-velocity motion in the x-direction and constant-acceleration motion in the y-direction:

$$a_x = 0$$

$$a_y = -g$$

Try to memorize these formulas, but you must know how to derive them

- Time of travel $t = \frac{2v_0 \sin \theta}{g}$
- Maximum height of projectile $h = \frac{v_0^2 \sin^2 \theta}{2g}$
- Range of projectile $R = \frac{2v_0^2 \sin \theta \cos \theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$

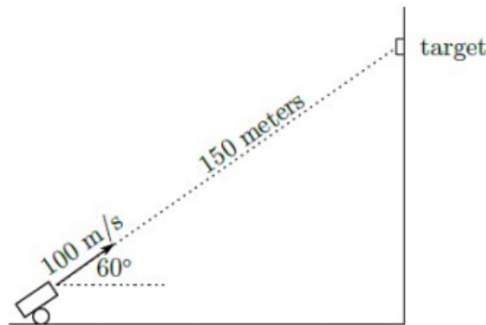
Other Useful Facts: (Bonus: Prove that these are true!)

1. The maximum range of a projectile is achieved at a projection angle of 45° .
2. The range of the projectile is the same for any two projection angles that sum to 90° .

Practice Problems

1. Two students are on a balcony 25.6 m above the street. One student throws a ball vertically downward at 15.7 m/s; at the same instant, the other student throws a second ball vertically upward at the same speed. The second ball just misses the balcony on the way down. The first ball hits the ground with a perfectly inelastic collision.
 - (a) What is the difference in the two balls' time in the air?
 - (b) What is the velocity of each ball as it strikes the ground?
 - (c) How far apart are the balls 0.800 s after they are thrown?
 - (d) How far apart are the balls 1.80 s after they are thrown?

5. A 470 gram lead ball is launched at a 60 degree angle above the horizontal with an initial speed of 100 m/s directly toward a target on a vertical cliff wall that is 150 meters away as shown in the figure.

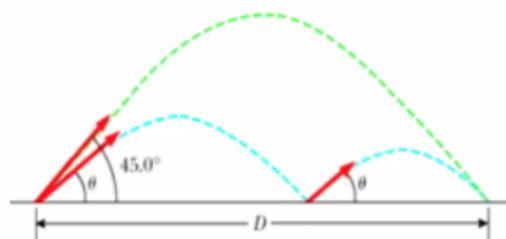


Ignoring air friction, by what distance does the lead ball miss the target when it hits the cliff wall?

- (A) 1.3 m
- (B) 2.2 m
- (C) 5.0 m
- (D) 7.1 m
- (E) 11 m

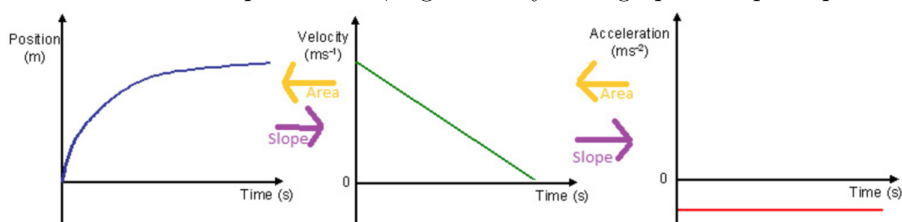
A cannon A is at the edge of a cliff with a 800 m drop. Cannon B is on the ground below the cliff and 600 m horizontally away from it. Cannon A shoots a cannonball directly towards cannon B at 60 m/s . Cannon B shoots a cannonball directly towards cannon A at 40 m/s. Will the two cannonballs hit each other in midair?

3. When baseball outfielders throw the ball, they usually allow it to take one bounce, on the theory that the ball arrives at its target sooner that way. Suppose that, after the bounce, the ball rebounds at the same angle θ that it has when it was released (as shown in the figure below), but loses $1/3$ of its speed.
- (a) Assuming that the ball is always thrown with the same initial speed, at what angle θ should the ball be thrown in order to go the same distance D with one bounce as a ball thrown upward at 45.0° with no bounce?
- (b) Determine the ratio of the times for the one-bounce and no-bounce throws.



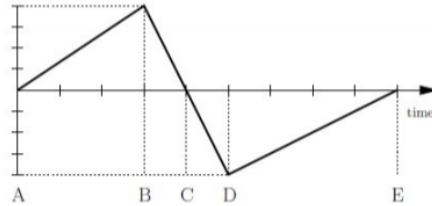
IV. Graphing:

Use facts about slope and area, e.g. velocity time graph is slope of position time graph.



Practice Problems

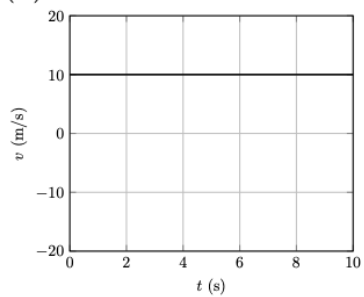
Questions 1 to 3 refer to the figure below which shows a representation of the motion of a squirrel as it runs in a straight-line along a telephone wire. The letters A through E refer to the indicated times.



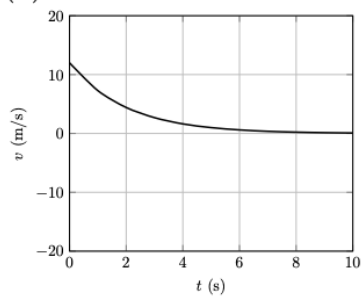
1. If the graph is a graph of POSITION vs. TIME, then the squirrel has the greatest speed at what time(s) or during what time interval(s)?
 - (A) From A to B
 - (B) From B to C only
 - (C) From B to D
 - (D) From C to D only
 - (E) From D to E
2. If, instead, the graph is a graph of VELOCITY vs. TIME, then the squirrel has the greatest speed at what time(s) or during what time interval(s)?
 - (A) at B
 - (B) at C
 - (C) at D
 - (D) at both B and D
 - (E) From C to D
3. If, instead, the graph is a graph of ACCELERATION vs. TIME and the squirrel starts from rest, then the squirrel has the greatest speed at what time(s) or during what time interval?
 - (A) at B
 - (B) at C
 - (C) at D
 - (D) at both B and D
 - (E) From C to D

1. Which of the following graphs best shows the velocity versus time of an object originally moving upward in the presence of air friction?

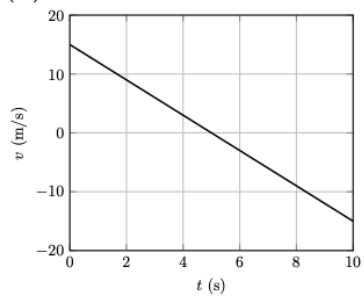
(A)



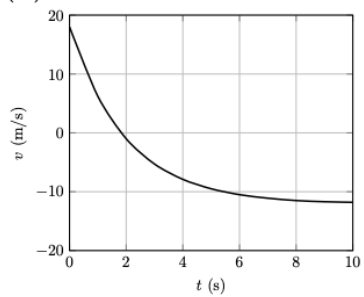
(B)



(C)



(D)



(E)

