



Fluids

January 29, 2026

Introduction

Density and Pressure

Density is defined as:

$$\rho = \frac{M}{V}$$

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Pressure is defined as:

$$P = \frac{F}{A}$$

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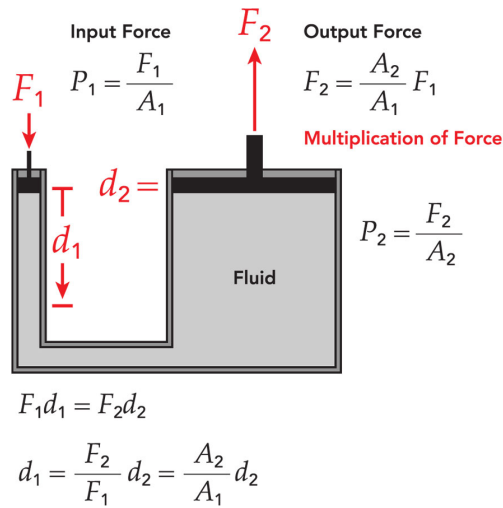
Both are important concepts to the discussion of fluids.

Pascal's Law: For an incompressible fluid, a pressure change at any point is transmitted equally throughout the fluid.

At two points in a fluid column, we have

$$\Delta P = \rho g \Delta h.$$

- When the fluid is in equilibrium, the pressure of the fluid is equal at any given height.
- Atmospheric pressure is P_{atm} or P_0
- **Gauge Pressure** is the pressure relative to atmospheric pressure, equal to $P - P_0$
- To get the total change in pressure between two points, you can add up the changes in pressure between those two points

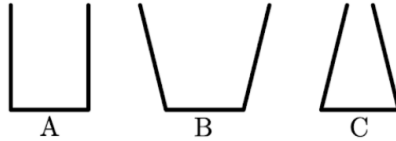


Practice Problems

The small piston of a hydraulic lift has an area of 0.20 m^2 . A car weighing $1.2 \times 10^4 \text{ N}$ sits on a rack mounted on the large piston. The large piston has an area of 0.90 m^2 . How large force must be applied to the small piston to support the car?

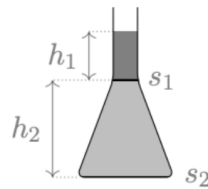


Flasks A, B, and C each have a circular base with a radius of 2 cm. An equal volume of water is poured into each flask, and none overflow. Rank the force of water F on the base of the flask from greatest to least.



- (A) $F_A > F_B > F_C$
- (B) $F_A > F_C > F_B$
- (C) $F_B > F_C > F_A$
- (D) $F_C > F_A > F_B$
- (E) $F_A = F_B = F_C$

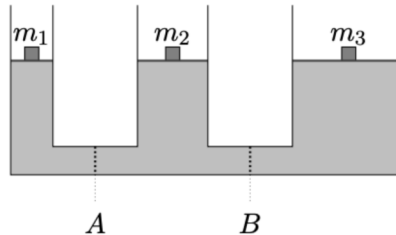
As shown in the figure, a vessel contains two types of liquid: the liquid with density ρ_1 on top and ρ_2 on the bottom. The depth of the top liquid is h_1 , and the interface area between the top and the bottom liquid is s_1 . The bottom liquid has a depth of h_2 . The area of the bottom of the vessel is s_2 . What is the gauge pressure (i.e. pressure in excess of atmospheric pressure) at the bottom of the vessel?



- (A) $(\rho_1 h_1 + \rho_2 h_2)g$
- (B) $\frac{\rho_1 s_1 h_1 + \rho_2 s_2 h_2}{s_2} g$
- (C) $\frac{1}{2}(\rho_1 + \rho_2)(h_1 + h_2)g$
- (D) $(\rho_1 + \rho_2)(h_1 + h_2)g$
- (E) $\rho_2(h_1 + h_2)g$

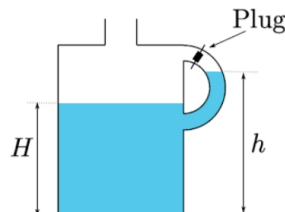


In the device shown, blocks of various masses are placed on pistons so that the device is in equilibrium. (The fluid in the drawing is to scale.) There are valves that are both initially open at locations A and B . One of the valves is closed, and the system is allowed to come to equilibrium. How will this affect the height of the mass m_2 ?



- (A) Closing either valve will have no effect on the height of m_2 .
- (B) Closing either valve causes m_2 to rise.
- (C) Closing either valve causes m_2 to fall.
- (D) Closing valve A causes m_2 to rise, and closing valve B causes m_2 to fall.
- (E) Closing valve A causes m_2 to fall, and closing valve B causes m_2 to rise.

The handle of a gallon of milk is plugged by a manufacturing defect. After removing the cap and pouring out some milk, the level of milk in the main part of the jug is lower than in the handle, as shown in the figure. Which statement is true of the gauge pressure P of the milk at the bottom of the jug? ρ is the density of the milk.



- (A) $P = \rho gh$
- (B) $P = \rho gH$
- (C) $\rho gH < P < \rho gh$
- (D) $P > \rho gh$
- (E) $P < \rho gH$



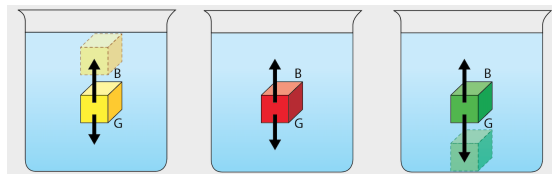
Archimedes' Principle: Any object immersed in a fluid will experience an upward buoyant force equal to the weight of the displaced fluid:

$$F_B = \rho V g,$$

where ρ is the density of the fluid, and V is the volume of the displaced liquid.

- For an object on the surface, use the volume that is submerged to calculate buoyant force.
- An object immersed will usually have 2 forces: gravity mg and buoyant force $\rho V g$.
- For an object floating in equilibrium, these two forces balance.
- One helpful note is that when solving problems dealing with accelerated reference frames with fluids, the buoyancy force is equivalent to

$$F_B = \rho V g_{\text{eff}}$$





Practice Problems

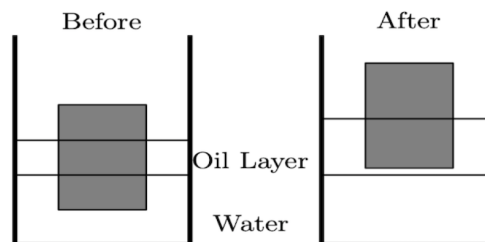
When a block of wood with a weight of 30 N is completely submerged under water the buoyant force on the block of wood from the water is 50 N. When the block is released it floats at the surface. What fraction of the block will then be visible above the surface of the water when the block is floating?

- (A) $1/15$
- (B) $1/5$
- (C) $1/3$
- (D) $2/5$
- (E) $3/5$

An object of uniform density floats partially submerged so that 20% of the object is above the water. A 3 N force presses down on the top of the object so that the object becomes fully submerged. What is the volume of the object? The density of water is $\rho_{\text{H}_2\text{O}} = 1000 \text{ kg/m}^3$.

- (A) $V_{\text{object}} = 0.3 \text{ L}$
- (B) $V_{\text{object}} = 0.67 \text{ L}$
- (C) $V_{\text{object}} = 1.2 \text{ L}$
- (D) $V_{\text{object}} = 1.5 \text{ L}$
- (E) $V_{\text{object}} = 3.0 \text{ L}$

A 3.0 cm thick layer of oil with density $\rho_o = 800 \text{ kg/m}^3$ is floating above water that has density $\rho_w = 1000 \text{ kg/m}^3$. A solid cylinder is floating so that $1/3$ is in the water, $1/3$ is in the oil, and $1/3$ is in the air. Additional oil is added until the cylinder is floating only in oil. What fraction of the cylinder is in the oil?



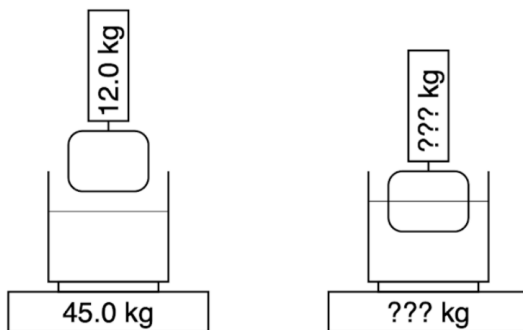
- (A) $3/5$
- (B) $3/4$
- (C) $2/3$
- (D) $8/9$
- (E) $4/5$



A sphere floats in water with $2/3$ of the volume of the sphere submerged. The sphere is removed and placed in oil that has $3/4$ the density of water. If it floats in the oil, what fraction of the sphere would be submerged in the oil?

- (A) $1/12$
- (B) $1/2$
- (C) $8/9$
- (D) $17/12$
- (E) The sphere will not float, it will sink in the oil.

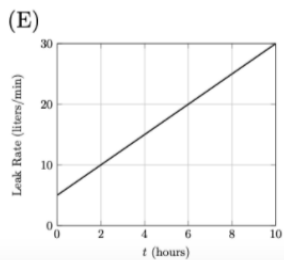
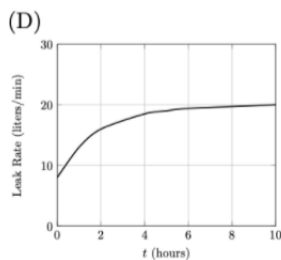
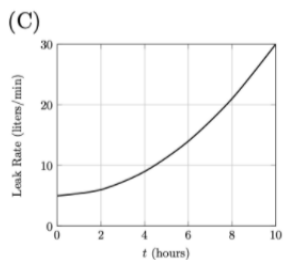
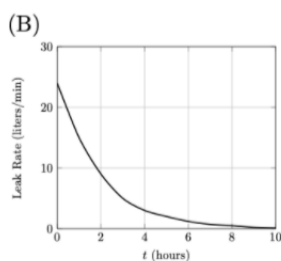
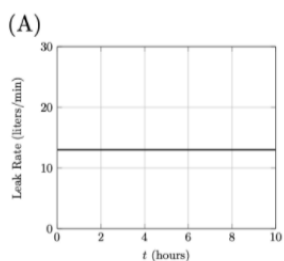
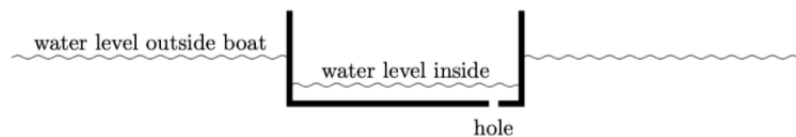
A container of water is sitting on a scale. Originally, the scale reads $M_1 = 45$ kg. A block of wood is suspended from a second scale; originally the scale read $M_2 = 12$ kg. The density of wood is 0.60 g/cm^3 ; the density of the water is 1.00 g/cm^3 . The block of wood is lowered into the water until half of the block is beneath the surface. What is the resulting reading on the scales?



- (A) $M_1 = 45$ kg and $M_2 = 2$ kg.
- (B) $M_1 = 45$ kg and $M_2 = 6$ kg.
- (C) $M_1 = 45$ kg and $M_2 = 10$ kg.
- (D) $M_1 = 55$ kg and $M_2 = 6$ kg.
- (E) $M_1 = 55$ kg and $M_2 = 2$ kg.



22. A small hole is punched into the bottom of a rectangular boat, allowing water to enter the boat. As the boat sinks into the water, which of the following graphs best shows how the rate water flows through the hole varies with time? Assume that the boat remains horizontal as it sinks.





Bonus Problems

A block floats partially submerged in a container of liquid. When the entire container is accelerated upward, which of the following happens? Assume that both the liquid and the block are incompressible.

- (A) The block descends down lower into the liquid.
- (B) The block ascends up higher in the liquid.
- (C) The block does not ascend nor descend in the liquid.
- (D) The answer depends on the direction of motion of the container.
- (E) The answer depends on the rate of change of the acceleration

A trough half-filled with water is suspended from wires as shown. The tension is initially the same in each wire.



A boat is placed in the trough directly under the left wire. It floats without touching the sides of the trough or overflowing the water. How does the tension in the wires change as a result?

- (A) The tension is not affected in either wire.
- (B) The tension increases equally in both wires.
- (C) The tension increases in the left wire and decreases in the right wire.
- (D) The tension increases in the left wire and stays the same in the right wire.
- (E) The tension increases in the left wire and increases by a smaller amount in the right wire.



Continuity Equation: For an incompressible fluid, the volume of some region of fluid stays constant (conservation of mass), so the flux or volume flow rate

$$J = \frac{dV}{dt} = \frac{d(Ax)}{dt} = A \frac{dx}{dt} = Av$$

is constant. The conservation can be written as:

$$A_1 v_1 = A_2 v_2.$$

Bernoulli's Equation: A statement of conservation of energy, where the work done by a change in pressure causes a change in potential and kinetic energy of a fluid.

$$W_1 + \frac{1}{2}mv_1^2 + mgh_1 = W_2 + \frac{1}{2}mv_2^2 + mgh_2$$

The work from done by pressure is $W = Fd = PAd = PV$. Thus, dividing by volume V , we get

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2.$$

- For a fluid exposed to air/atmosphere, the pressure is P_{atm} .



Practice Problems

A large tank of water, open at the top, has a small hole through its side a distance h below the surface of the water. Find the speed of the water as it flows out the hole.

Two streams of water flow through the U-shaped tubes shown. The tube on the left has cross-sectional area A , and the speed of the water flowing through it is v ; the tube on the right has cross-sectional area $A' = 1/2A$. If the net force on the tube assembly is zero, what must be the speed v' of the water flowing through the tube on the right? Neglect gravity, and assume that the speed of the water in each tube is the same upon entry and exit.



- (A) $1/2v$
- (B) v
- (C) $\sqrt{2}v$
- (D) $2v$
- (E) $4v$