

Dynamics

September 15, 2025

Newton's Laws

- **1st Law** (Law of Inertia): An object in rest remains in rest unless acted upon by a force. An object in uniform motion remains in uniform motion unless acted upon by a force.
- **2nd Law**: The acceleration of an object is proportional to the net force acted on the object and inversely proportional to its mass. Symbolically, $a = \frac{F}{m}$ or $F = ma$.
- **3rd Law**: For every action, there is an equal and opposite reaction. If object A exerts a force on object B , then object B exerts a force on object A equal in magnitude and opposite in direction.

Forces

- A **force** is a push or pull exerted on an object that changes its motion.
- A force is a vector (magnitude + direction).
- Newton's 2nd Law relates the force on an object to its acceleration:

$$F_{net} = ma.$$

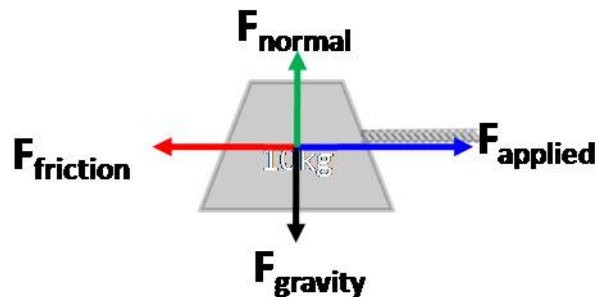
- The net force F_{net} on an object is the vector sum of all the forces acting on it.
- SI unit is Newton (N). One Newton is the force required to accelerate 1 kg of mass by 1 m/s².
- Examples of forces: Gravity, electromagnetic, normal force, friction, springs, tension
- Contact forces require contact to occur, while long-range forces (like gravity) act from a distance. (Microscopically, contact forces are caused by molecular bonds and electrostatic forces in atoms)

Common Forces

- An **applied force** is an external force applied to an object by a person or object, such as pushing or pulling.
- **Gravity** is a force that acts on any two objects with mass, causing them to attract. On Earth, gravity causes the **weight** force, which always points downwards with magnitude mg , where $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration on earth.
- The **normal force** is the contact force perpendicular (normal) to a surface.
- The **friction force** is a resisting force parallel to a surface. It acts in the opposite direction of motion. **Kinetic friction** acts when an object slides across a surface, while **static friction** occurs when an object is "stuck" on a surface, preventing its motion.
- The **tension force** is a force exerted by a rope or wire pulling on an object
- **Drag** is resistance through a fluid, such as air or water resistance. Like friction, it acts opposite the direction of motion.
- The **spring force** is a restoring force acting on an object at the end of a spring. For ideal Hookian springs, the restoring force is proportional to the displacement from the spring's rest length. $F = -kx$

Free Body Diagrams

- A **free body diagram** (FBD) is a diagram showing all the forces acting upon an object.
- Forces are represented as arrows starting from the point of action and in the direction of the force. Sometimes, the length of the arrows indicate the relative magnitude of the forces.



- Draw free body diagrams for the following scenarios:
 1. A book is at rest on a tabletop.
 2. An egg is free-falling from a nest in a tree. Neglect air resistance.
 3. A rightward force is applied to a book in order to move it across a desk with a rightward acceleration. Consider frictional forces.

4. A skydiver is descending with a constant velocity. Consider air resistance.
5. A football is moving upwards towards its peak after having been booted by the punter. Neglect air resistance.
6. A skier is being towed up a snow-covered hill by a tow rope.

Statics: Equilibrium

- An object in **equilibrium** is not accelerating; $a = 0$.
- An object in **static equilibrium** is not moving; $v = 0$.
- An object in **dynamic equilibrium** is moving at constant velocity.
- By Newton's 2nd Law, $a = 0$ implies $F = 0$. This can be broken up component-wise:

$$F_{net,x} = \sum F_x = ma_x = 0,$$

$$F_{net,y} = \sum F_y = ma_y = 0$$

must be true for an object in equilibrium.

- If an object is not in equilibrium, by Newton's 2nd Law:

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y.$$

- We will next examine how to apply forces to determine the motion of an object.

Weight

- **Weight** is the gravitational force exerted on a mass, $w = mg$.
- **Apparent Weight** is the force an object feels, due to contact forces supporting it. Often, this is the normal force.
- Anjay's mass is 70 kg. He is standing on a scale in an elevator that is moving at 5.0 m/s. As the elevator slows to a stop, the scale reads 750 N. Before it stopped, was the elevator moving up or down? How long did the elevator take to come to rest?
- **Weightlessness** occurs when an object experiences 0 apparent weight.
- What is the apparent weight of an object in freefall?

Normal Forces

- A **normal force** acts perpendicular, and adjusts so that an object stays on the surface without penetrating through it.
- Find the normal force acting on a block of mass m on a table.
- Find the normal force of a block of mass m on a frictionless incline with angle θ .

Friction

- **Static Friction** is the force a surface exerts to prevent the object from slipping. The maximum static friction between 2 surfaces is given by

$$f_{s,max} = \mu_s N,$$

where μ_s is the **coefficient of static friction**.

- The direction of static friction is such as to oppose motion.
 - The magnitude f_s of static friction adjusts itself so that the net force is zero and the object doesn't move.
 - The magnitude of static friction cannot exceed the maximum value. If the friction force needed to keep the object stationary is greater than $f_{s,max}$, the object slips and starts to move.
- **Kinetic Friction** is the force a surface exerts when an object is slipping. Kinetic friction has a *constant* magnitude given by

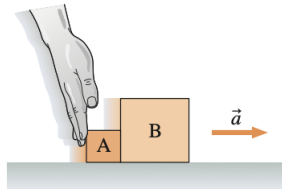
$$f_k = \mu_k N,$$

where μ_k is the **coefficient of kinetic friction**. The direction opposes the motion, and the magnitude is constant (does not depend on velocity).

- Carol wants to move her 32 kg sofa to a different room in the house. She places “sofa sliders,” slippery disks with $\mu_k = 0.080$ under the feet of the sofa. She then pushes the sofa at a steady 0.40 m/s across the floor. How much force does she apply to the sofa?
- A 50.0 kg steel file cabinet is in the back of a dump truck. The truck's bed, also made of steel, is slowly tilted. What is the magnitude of the static friction force on the cabinet when the bed is tilted 20°? At what angle will the file cabinet begin to slide?

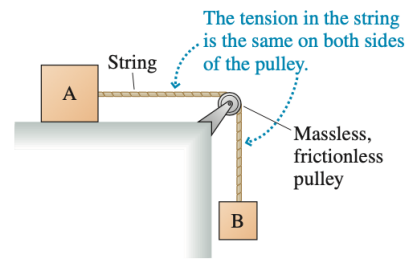
Interacting Objects

- Use Newton's 3rd Law.
- Objects in contact, moving together, must have same velocity and acceleration.
- A 5.0 kg block A is being pushed with a 3.0 N force on a frictionless surface. In front of this block is a 10 kg block B; the two blocks move together. What force does block A exert on block B?



Ropes and Pulleys

- When a rope is pulled, it transmits a **tension** force across the rope. Assuming the rope is massless, the tension is constant throughout the rope.
- For an ideal pulley (massless, frictionless), the tension of the rope on both sides of the pulley is the same.
- A 5.0 kg block B is hanging from the side of a frictionless table, connected by a pulley to

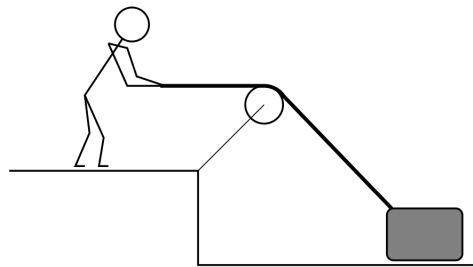


a 10 kg block A. What is the acceleration of block A?

- Now suppose the both blocks are hanging from opposite sides of a pulley. What is the acceleration of block A?

Practice Problems

11. As shown below, Lily is using the rope through a fixed pulley to move a box with constant speed v . The kinetic friction coefficient between the box and the ground is $\mu < 1$; assume that the fixed pulley is massless and there is no friction between the rope and the fixed pulley. Then, while the box is moving, which of the following statements is correct?

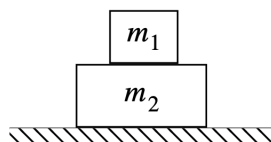


- (A) The magnitude of the force on the rope is constant.
- (B) The magnitude of friction between the ground and the box is decreasing.
- (C) The magnitude of the normal force of the ground on the box is increasing.
- (D) The pressure of the box on the ground is increasing.
- (E) The pressure of the box on the ground is constant.

An experiment consists of pulling a heavy wooden block across a level surface with a spring force meter. The constant force for each try is recorded, as is the acceleration of the block. The data are shown below.

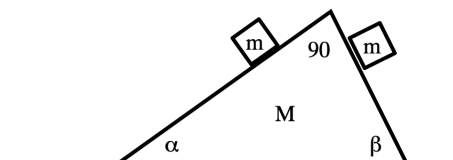
Force F in Newtons	3.05	3.45	4.05	4.45	5.05
acceleration a in meters/second ²	0.095	0.205	0.295	0.405	0.495

10. Which is the best value for the mass of the block?
- (a) 3 kg
 - (b) 5 kg
 - (c) 10 kg
 - (d) 20 kg
 - (e) 30 kg
11. Which is the best value for the coefficient of friction between the block and the surface?
- (a) 0.05
 - (b) 0.07
 - (c) 0.09
 - (d) 0.5
 - (e) 0.6
13. Two blocks of masses $m_1 = 2.0$ kg and $m_2 = 1.0$ kg are stacked together on top of a frictionless table as shown. The coefficient of static friction between the blocks is $\mu_s = 0.20$. What is the minimum horizontal force that must be applied to the top block to make it slide across the bottom block?



- (A) 4.0 N
- (B) 6.0 N
- (C) 8.0 N
- (D) 12.0 N
- (E) The top block will not slide across the bottom block

11. A right-triangular wooden block of mass M is at rest on a table, as shown in figure. Two smaller wooden cubes, both with mass m , initially rest on the two sides of the larger block. As all contact surfaces are frictionless, the smaller cubes start sliding down the larger block while the block remains at rest. What is the normal force from the system to the table?



- (A) $2mg$
 (B) $2mg + Mg$
 (C) $mg + Mg$
 (D) $Mg + mg(\sin \alpha + \sin \beta)$
 (E) $Mg + mg(\cos \alpha + \cos \beta)$

Springs

Most of the time, we consider a linear spring that follows Hooke's Law.

Hooke's Law: The retarding force of a spring is proportional to its displacement from equilibrium

$$F = -kx.$$

- Here we use $x = \Delta x$ as displacement from equilibrium (rest length), not position.
- Remember that by Newton's 3rd law, the force of a (massless) spring on both sides must be equal.

The potential energy of the spring is the work done to expand it:

$$U = W = \int F \cdot dx = \frac{1}{2}kx^2.$$

Practice Problems

A spring has an equilibrium length of 2.0 meters and a spring constant of 10 newtons/meter. Alice is pulling on one end of the spring with a force of 3.0 newtons. Bob is pulling on the opposite end of the spring with a force of 3.0 newtons, in the opposite direction. What is the resulting length of the spring?

- (A) 1.7 m
 (B) 2.0 m
 (C) 2.3 m
 (D) 2.6 m
 (E) 8.0 m

A block of mass $m = 3.0$ kg is moving on a horizontal surface towards a massless spring with spring constant $k = 80.0$ N/m. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.50$. The block has a speed of 2.0 m/s when it first comes in contact with the spring. How far will the spring be compressed?

- (A) 0.19 m
- (B) 0.24 m
- (C) 0.39 m
- (D) 0.40 m
- (E) 0.61 m

Two equal masses m are connected by an elastic string that acts like an ideal spring with spring constant k and unstretched length l . The two masses are hung over a frictionless pulley. What is the total length of the string at equilibrium? (Diagram not necessarily to scale.)



- (A) $2mg/k$
- (B) $l + mg/k$
- (C) $l + mg/2k$
- (D) $l + 2mg/k$
- (E) There is not enough information to decide.

Three point masses m are attached together by identical springs. When placed at rest on a horizontal surface the masses form a triangle with side length l . When the assembly is rotated about its center at angular velocity ω , the masses form a triangle with side length $2l$. What is the spring constant k of the springs?

- (A) $2m\omega^2$
- (B) $\frac{2}{\sqrt{3}}m\omega^2$
- (C) $\frac{2}{3}m\omega^2$
- (D) $\frac{1}{\sqrt{3}}m\omega^2$
- (E) $\frac{1}{3}m\omega^2$

Vertical Springs

A spring under the influence of gravity acts equivalently to normal spring, except its equilibrium length is displaced.

Consider a vertical spring of rest length l_0 with one end attached to the ground and the other end attached to a block of mass m . The force on the block is

$$F = -k(x - l_0) - mg = -k\left(x - \left(l_0 - \frac{mg}{k}\right)\right)$$

Or, equilibrium occurs when

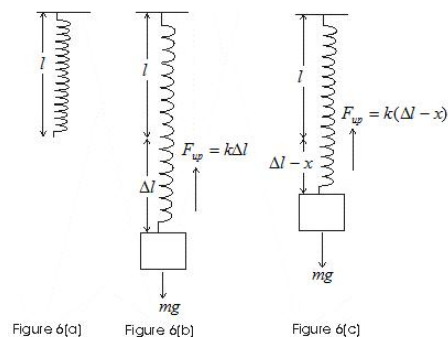
$$k\Delta x = mg \implies \Delta x = \frac{mg}{k}.$$

Thus, it is equivalent to a spring **without gravity** with spring constant k and rest length $l_0 - \frac{mg}{k}$.

What about a vertical spring with one end attached to the ceiling instead? Then, the force on the block is

$$F = -k(x - l_0) + mg = -k\left(x - \left(l_0 + \frac{mg}{k}\right)\right)$$

So it is equivalent to a spring **without gravity** with spring constant k and rest length $l_0 + \frac{mg}{k}$.



- Be careful not to include gravity again when using this model, otherwise you are applying gravity twice.
- Often the situation won't exactly resemble this model, so it's more important to understand how we arrived at this conclusion.

Practice Problems

The following information applies to questions 23 and 24

A man with mass m jumps off of a high bridge with a bungee cord attached to his ankles. The man falls through a maximum distance H at which point the bungee cord brings him to a momentary rest before he bounces back up. The bungee cord is perfectly elastic, obeying Hooke's force law with a spring constant k , and stretches from an original length of L_0 to a final length $L = L_0 + h$. The maximum tension in the Bungee cord is four times the weight of the man.

23. Determine the spring constant k .

- (A) $k = \frac{mg}{h}$
- (B) $k = \frac{2mg}{h}$
- (C) $k = \frac{mg}{H}$
- (D) $k = \frac{4mg}{H}$
- (E) $k = \frac{8mg}{H}$

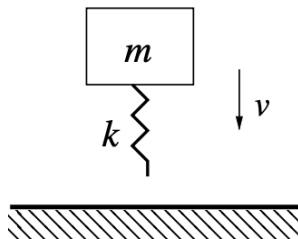
24. Find the maximum extension of the bungee cord h .

- (A) $h = \frac{1}{2}H$
- (B) $h = \frac{1}{4}H$
- (C) $h = \frac{1}{5}H$
- (D) $h = \frac{2}{5}H$
- (E) $h = \frac{1}{8}H$

A 2.0 kg object falls from rest a distance of 5.0 meters onto a 6.0 kg object that is supported by a vertical massless spring with spring constant $k = 72 \text{ N/m}$. The two objects stick together after the collision, which results in the mass/spring system oscillating. What is the maximum magnitude of the displacement of the 6.0 kg object from its original location before it is struck by the falling object?

- (A) 0.27 m
- (B) 1.1 m
- (C) 2.5 m
- (D) 2.8 m
- (E) 3.1 m

A pogo stick is modeled as a massless spring of spring constant k attached to the bottom of a block of mass m . The pogo stick is dropped with the spring pointing downward and hits the ground with speed v . At the moment of the collision, the free end of the spring sticks permanently to the ground.



During the subsequent oscillations, the maximum speed of the block is

- (A) v
- (B) $v + 2mg^2/kv$
- (C) $v + mg^2/kv$
- (D) $\sqrt{v^2 + 2mg^2/k}$
- (E) $\sqrt{v^2 + mg^2/k}$

Combining Springs

Remember that the spring constant is defined as $k = F/x$. When multiple springs are put together, you can determine the spring constant by computing the displacement x when some force F is applied to one end.

Springs in Parallel: For n springs with same equilibrium length, the equivalent spring constant is the sum of the individual spring constants

$$k' = k_1 + k_2 + \cdots + k_n.$$

Proof:

Consider an object connected to two springs in parallel, with spring constants k_1 and k_2 . Then, the net force is

$$F = k_1x + k_2x = (k_1 + k_2)x.$$

Thus, the equivalent spring constant is

$$k' = k_1 + k_2.$$

Springs in Series: For n springs (not necessarily same equilibrium length) connected in series, the reciprocal of the equivalent spring constant is the sum of the reciprocals of the individual spring constants.

$$\frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_n}.$$

Proof:

Consider two springs with spring constants k_1 and k_2 , stretched by a displacement x . The connection point between the two springs must be in equilibrium, since it's not moving, so the force from both springs must be equal. For some applied force F , the displacement (from equilibrium) is

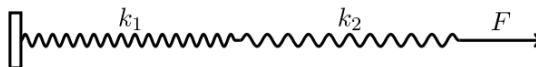
$$x = x_1 + x_2 \implies \frac{F}{k'} = \frac{F}{k_1} + \frac{F}{k_2} \implies \frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2}$$

Practice Problems

A spring stretched to double its unstretched length has a potential energy U_0 . If the spring is cut in half, and each half spring is stretched to double its unstretched length, then the total potential energy stored in the two half springs will be

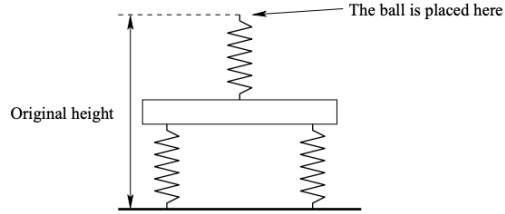
- (A) $4U_0$
- (B) $2U_0$
- (C) U_0
- (D) $U_0/2$
- (E) $U_0/4$

Two springs of spring constants k_1 and k_2 , respectively, are connected in series and stretched, as shown below. What is the ratio of their potential energies, U_1/U_2 ?



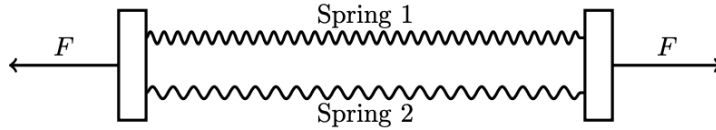
- (A) 1
- (B) k_1/k_2
- (C) k_2/k_1
- (D) $(k_1/k_2)^2$
- (E) $(k_2/k_1)^2$

- . A spring system is set up as follows: a platform with a weight of 10 N is on top of two springs, each with spring constant 75N/m. On top of the platform is a third spring with spring constant 75N/m. If a ball with a weight of 5.0 N is then fastened to the top of the third spring and then slowly lowered, by how much does the height of the spring system change?



- (A) 0.033 m
- (B) 0.067 m
- (C) 0.100 m
- (D) 0.133 m
- (E) 0.600 m

A spring of relaxed length ℓ_1 and spring constant k_1 is placed ‘in parallel’ with a spring of relaxed length ℓ_2 and spring constant k_2 . A force F is applied to each end.



The combination of the springs acts like a single spring with spring constant k and relaxed length ℓ where

- (A) $k = k_1 + k_2$ and $\ell = \ell_1 \ell_2 / (\ell_1 + \ell_2)$
- (B) $k = k_1 + k_2$ and $\ell = (\ell_1 k_1 + \ell_2 k_2) / (k_1 + k_2)$
- (C) $k = k_1 + k_2$ and $\ell = (\ell_1 k_2 + \ell_2 k_1) / (k_1 + k_2)$
- (D) $k = (\ell_1 k_1 + \ell_2 k_2) / (\ell_1 + \ell_2)$ and $\ell = (\ell_1 k_1 + \ell_2 k_2) / (k_1 + k_2)$
- (E) $k = (\ell_2 k_1 + \ell_1 k_2) / (\ell_1 + \ell_2)$ and $\ell = (\ell_1 k_2 + \ell_2 k_1) / (k_1 + k_2)$

Zero-length Springs

For an ideal spring with zero natural length, the force is proportional to its length

$$\mathbf{F} = k\mathbf{r}$$

A special result is that the x-component and y-component forces are proportional to their respective displacements:

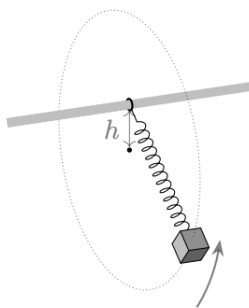
$$F_x = kr \cos \theta = kr \frac{x}{r} = kx$$

and

$$F_y = kr \sin \theta = kr \frac{y}{r} = ky$$

Practice Problems

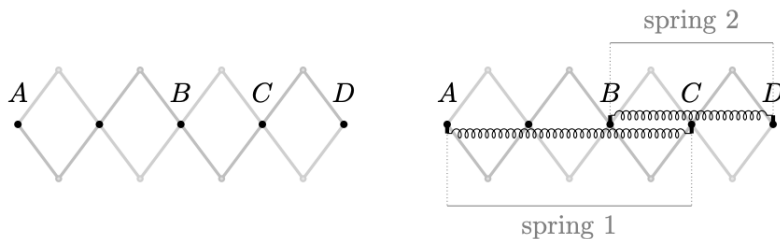
A mass m is connected to one end of a zero-length spring with spring constant k . The other end of the spring is connected to a frictionless bearing mounted around a horizontal pole so that the mass can swing in a vertical circle of radius R around the pole. The setup is shown in the figure below. What is the vertical distance h between the center of the circular orbit and the axis of the pole? Assume that both the diameter of the pole and the rest length of the spring are negligible compared to R .



- (A) $\sqrt{mgR/k}$
- (B) $R\sqrt{(R + mg/k)/(R - mg/k)}$
- (C) $R - mg/k$
- (D) mg/k
- (E) $\sqrt{R^2 - (mg/k)^2}$

Extra Problems

An extendable arm is made from rigid beams free to pivot around the dots shown. Spring 1, with equilibrium length L_1 , is attached between points A and C while spring 2, with equilibrium length L_2 is attached between B and D. The system is allowed to come to equilibrium. In equilibrium, what is the ratio of the tension in spring 1 to the tension in spring 2?



(A) $-2/3$

(B) $-3/4$

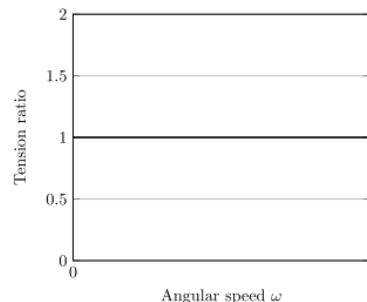
(C) -1

(D) $-3/2$

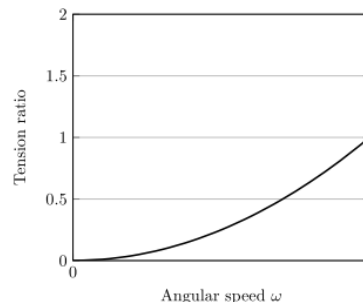
(E) There is not enough information to determine the ratio.

A vertical pole has two massless strings, both of length L , attached a distance L apart. The other ends of the strings are attached to a mass M . The mass is rotated around the pole with an angular speed ω . Which of the following graphs best gives the ratio of the tension in the bottom string to the tension in the top string as a function of ω ?

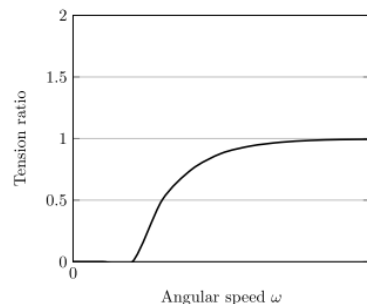
(A)



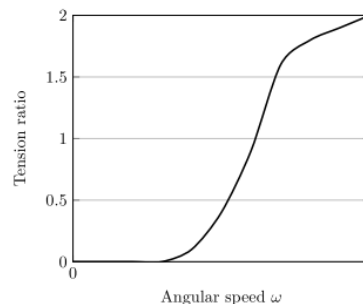
(B)



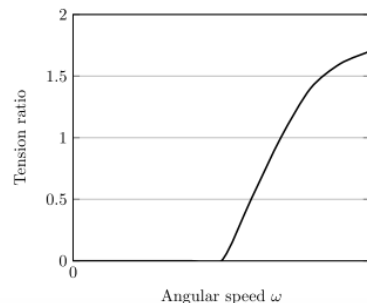
(C)



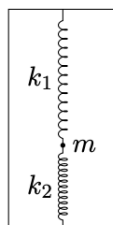
(D)



(E)



A ball of negligible radius and mass m is connected to two ideal springs. Each spring has rest length ℓ_0 . The springs are connected to the ball inside a box of height $2\ell_0$, and the ball is allowed to come to equilibrium, as shown. Under what condition is this equilibrium point stable with respect to small horizontal displacements?



(A) $k_1 > k_2$

(B) $k_2 > k_1$

(C) $k_1 - k_2 > mg/\ell_0$

(D) $k_1 k_2 / (k_1 + k_2) > mg/\ell_0$

(E) $k_1 k_2 / (k_1 - k_2) > mg/\ell_0$